

New look at the QCD factorization

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We show that both the k_T - and collinear factorization for DIS structure functions can be obtained by consecutive reductions of the Compton scattering amplitude. Each of these reductions is an approximation valid under certain assumptions. In particular, the transitions to the k_T - factorization is possible when the virtualities of the partons connecting the perturbative and non-perturbative blobs are space-like. Then, if the parton distribution has a sharp maximum in k_\perp , the k_T factorization can be reduced to the collinear factorization.

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I. INTRODUCTION

The QCD factorization is the fundamental concept to provide theoretical grounds for applying the Perturbative QCD to description of hadronic reactions. According to the factorization, any scattering amplitude A in QCD can be represented as a convolution of a perturbative (E) and non-perturbative (T) contributions:

$$A = E \otimes T \quad (1)$$

There are two kinds of the factorization in the literature: Collinear factorization[1] and the k_T - factorization[2] where the DIS structure functions $f(x, Q^2)$ are respectively represented as follows:

$$f(x, Q^2) = \int_x^1 \frac{d\beta}{\beta} f^{(pert)}(x/\beta, Q^2/\mu^2) \phi(\beta, \mu^2) \quad (2)$$

and

$$f(x, Q^2) = \int_x^1 \frac{d\beta}{\beta} \int \frac{dk_\perp^2}{k_\perp^2} f^{(pert)}(x/\beta, Q^2/k_\perp^2) \Phi(\beta, k_\perp^2) \quad (3)$$

where $f^{(pert)}$ stand for the perturbative components of the structure functions; ϕ and Φ are the parton distributions and μ is the factorization scale. In what follows we obtain Eqs. (2,3), simplifying the factorized expression for the amplitude $A_{\mu\nu}$ of the Compton scattering off a hadron target. By doing so, we summarize and generalize the results obtained in [3]. Using appropriate projection operators P_r the Compton amplitude $A_{\mu\nu}$ can be expanded into a set of invariant amplitudes A_r . According to the Optical Theorem, every structure function f_r can be expressed through A_r :

$$f_r = \frac{1}{\pi} \Im A_r \quad (4)$$

Among amplitudes A_r there is the amplitude A_S related to the structure function F_1 singlet. We will address this amplitude as the singlet and will address as non-singlets to all other invariant amplitudes and use for them the generic notation A_{NS} . We also will use the generic notation A for both the singlet and non-singlet amplitudes when it is relevant.

II. BASIC FACTORIZATION FOR THE COMPTON AMPLITUDE

Let us expand the invariant amplitude A into a set of convolutions depicted in Fig. 1 where the t - channel states involve arbitrary number of partons.

Throughout the paper we will consider only the first graph in Fig. 1 where the blobs are connected by the two-parton state, with the partons being quarks. Consideration of the two-gluon state yields the same results as shown in [3]. All blobs in Fig. 1 can contain both perturbative and non-perturbative contribution, so this kind of factorization does not correspond to the conventional scenario of the QCD factorization. We will address it as the primary convolution. Introducing the Sudakov parametrization of the moment r :

$$k = -\alpha(q + xp) + \beta p + k_\perp, \quad (5)$$

we can write the primary convolution as follows, using the :

$$A(q^2, w) = \int_{-\infty}^{\infty} \frac{d\beta}{\beta} \int_0^{\infty} dk_\perp^2 \int_{-\infty}^{\infty} d\alpha \tilde{A}(w\beta, q^2, k^2) \frac{B}{(k^2)^2} T(w\alpha, k^2), \quad (6)$$

where \tilde{A} and T denote the upper and lower blobs respectively; $w = 2pq$, $k^2 = -w\alpha\beta - k_\perp^2$ and factor B , with $B = w(\alpha^2 + \beta^2) + k_\perp^2$, appears because of simplification of the spin structure of the intermediate quarks. We have skipped in Eq. (6) dependence on unessential arguments like masses, spin, etc. The integrand in Eq. (6) becomes singular at $k^2 \rightarrow 0$. This infrared (\equiv IR) divergence must be regulated. The IR-sensitive perturbative contents for the singlet and non-singlet amplitudes are different. A_{NS} contain the IR-sensitive perturbative logarithms whereas A_S includes both logarithms and the power-factor:

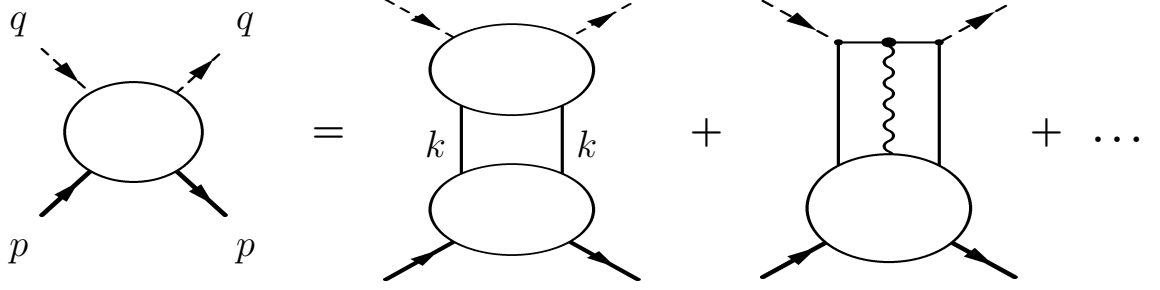


FIG. 1. Representation of $A_{\mu\nu}$ through the convolution of two blobs.

$$A_{NS} = A_{NS}(\ln(w\beta), \ln(Q^2/k^2)), \quad A_S = (w\beta/k^2) M_S(\ln(w\beta), \ln(Q^2/k^2)). \quad (7)$$

Therefore in order to keep the integral Eq. (6) IR stable, amplitudes T must obey the following restrictions at small k^2 :

$$T_{NS} \sim (k^2)^\gamma, \quad T_S \sim (k^2)^{1+\gamma}, \quad (8)$$

with $\gamma > 0$. Similarly, in order to get the ultraviolet stability of A the blob T at large α should decrease with growth of $|\alpha|$:

$$T_{NS} \sim |\alpha|^{-1-h}, \quad T_S \sim |\alpha|^{-h}. \quad (9)$$

Eq. (6) in the Born approximation is depicted in Fig. 2.

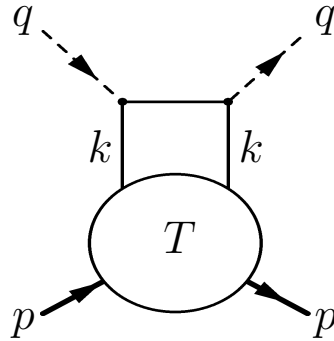


FIG. 2. Born approximation for the amplitude of the forward Compton scattering.

Radiative corrections are absent there, so blob T is totally non-perturbative. Inserting the radiative corrections into the Born approximation is depicted in Fig. 3.

We stress that we neglect graphs with extra propagators touching the lower blob (e.g. graph (b)) because they lead to the convolution with three or more intermediate partons depicted in Fig. 1 and we do not consider such multiparton

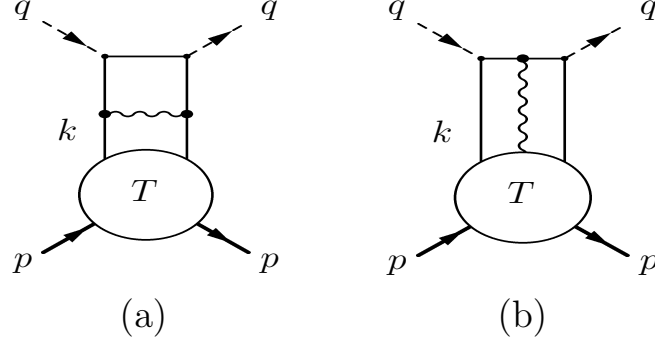


FIG. 3. Radiative corrections to the Born amplitude.

states in this paper. In order to back up this course of actions we would like to notice that all evolution equations available operate with the two-parton initial states only. So, we account for the graphs which do not touch it (e.g. graph (a)). Obviously all such graphs can be included into the upper blob, leaving the lower blob non-perturbative. As a result, we convert the convolution in Eq. (6) into the similarly looking convolution

$$A(q^2, w) = \int_{-\infty}^{\infty} \frac{d\beta}{\beta} \int_0^{\infty} dk_{\perp}^2 \int_{-\infty}^{\infty} d\alpha A^{(pert)}(w\beta, q^2, k^2) \frac{B}{(k^2)^2} T(w\alpha, k^2), \quad (10)$$

where the upper blob $A^{(pert)}$ is perturbative and the lower blob T is non-perturbative. The integral in (10) is free of IR singularities at small k^2 . Therefore, Eq. (10) corresponds to the concept of QCD factorization, though this factorization differs from the collinear and k_T -factorizations. By this reasons we will address it as the basic factorization. Applying Optical Theorem, we convert (10) into the basic factorization for the structure functions:

$$f(x, Q^2) = \int_{-\infty}^{\infty} \frac{d\beta}{\beta} \int_0^{\infty} dk_{\perp}^2 \int_{-\infty}^{\infty} d\alpha f^{(pert)}(x/\beta, Q^2/k^2) \frac{B}{(k^2)^2} \Psi(w\alpha, k^2) \quad (11)$$

where Ψ stands for the totally unintegrated parton distributions.

III. REDUCING BASIC FACTORIZATION TO k_T - AND COLLINEAR FACTORIZATIONS

In order to proceed from Eq. (11) to (3), we need to integrate out the α -dependence without touching the perturbative. Obviously, it cannot be done straightforwardly because $f^{(pert)}$ depends on α through k^2 . However, imposing the restriction

$$w\alpha\beta \ll k_{\perp}^2, \quad (12)$$

we can neglect this dependence in and integrate Ψ over α . As a result we arrive at (3) with

$$\Phi(\beta, k_{\perp}) = \int_{k_{\perp}^2/w}^{k_{\perp}^2/w\beta} d\alpha T(\alpha, k^2). \quad (13)$$

In order to keep (3) IR stable at $k_{\perp} \rightarrow 0$, the parton distributions Φ should decrease with k_{\perp} :

$$\Phi_{NS} \sim (k_{\perp}^2)^{\gamma}, \quad \Phi_S \sim (k_{\perp}^2)^{1+\gamma}. \quad (14)$$

Transition from the k_T -expression (3) to the collinear factorization (2) is also impossible in the straightforward way. Let us suppose that the k_{\perp} -dependence of $\Phi_{S,NS}$ in (3) has a peaked form with one or several sharp maximums. at $k_{\perp}^2 = \mu_0^2, \mu_1^2, \dots$ as shown in Fig. 4. We address such scales as intrinsic scales.

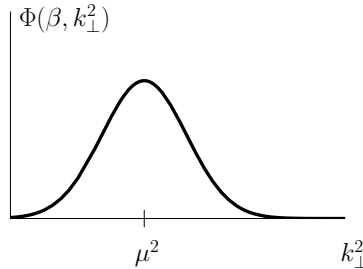


FIG. 4. The peaked form of $\Phi(\beta, k_\perp^2)$ with one maximum

We do not assume any special form for the curve in Fig. 3 save that it obeys the restriction (14). It allows us to approximately integrate over k_\perp in (3), dealing with Φ only and arriving at

$$f(x, Q^2) = \int_x^1 \frac{d\beta}{\beta} f^{(pert)}(x/\beta, Q^2/\mu_0^2) \varphi(\beta, \mu_0^2) \quad (15)$$

where the parton distributions φ are expressed through the distributions Φ which have been used in the k_T -factorization:

$$\varphi(\beta, \mu_0^2) = \int_0^w \frac{dk_\perp^2}{k_\perp^2} \Phi(\beta, k_\perp^2). \quad (16)$$

IV. COMPARISON OF CONVENTIONAL COLLINEAR FACTORIZATION AND EQ. (16).

The parton distribution ϕ in the conventional approach to the collinear factorization and distribution φ are widely different. The distribution ϕ includes both perturbative and non-perturbative contributions whereas φ is purely non-perturbative. The factorization scale μ used in the conventional approach is arbitrary while μ_0 corresponds to the maximum in Fig. 4. However, it is easy to relate them, using any kind of the perturbative evolution to evolve φ from scale μ_0 to μ . Naturally, the value of μ can be chosen anywhere between μ_0^2 and Q^2 . At the same time the perturbative part, $f^{(pert)}(x/\beta, Q^2/\mu_0^2)$, should be evolved from μ_0 to μ . As a result, we arrive at the conventional formula (2) where the convolution is independent of μ . In other words, changing the factorization scale from the intrinsic scale μ_0 to an arbitrary scale μ leads to the re-distribution of the radiative corrections between the upper and lower blobs of the collinear convolution. We do not specify which kind of the perturbative evolution should be used because our approach is insensitive to details of this evolution. In particular, the DGLAP equations can be used for such evolution.

V. RESTRICTIONS ON THE DGLAP FITS FOR THE PARTON DISTRIBUTIONS

Combining Eqs. (9, 13, 16) leads to the following dependence of the parton distributions Φ and φ at small β :

$$\Phi_{NS} \sim \beta^h, \quad \Phi_S \sim \beta^{-1+h}, \quad \varphi_{NS} \sim \beta^h, \quad \varphi_S \sim \beta^{-1+h}. \quad (17)$$

As shown in Eq. (18), the standard DGLAP -fits for the DIS structure functions in the collinear factorization include a normalization N , the singular factors x^{-a} , with $a > 0$, and the regular terms:

$$\delta q, \delta g = N x^{-a} (1-x)^b (1+cx^d), \quad (18)$$

where the parameters $N, a, b, c, d > 0$. Such expressions do not look as the ones obtained with the perturbative methods, so we identify them with non-perturbative distributions φ . Eq. (17) excludes the use of the singular factors

in the expressions for the non-singlet structure functions F_2, F_1^{NS}, g_1 , etc and also suppress the singular factors with $a > 1$ in the expressions for the singlet F_1 . However, the parton distributions used for F_1 and F_2 are identical, therefore the suppression of the singular factors with $a > 0$ can be applied to all structure functions, including the singlet F_1 . The singular factors x^{-a} in the DGLAP fits for initial parton densities should be removed from the fits because they contradict to the integrability of the basic convolutions of the Compton amplitudes.

VI. CONCLUSION

Both the k_T - and collinear factorizations are obtained by consecutive reductions of the Compton scattering amplitude represented as the convolution of two blobs connected by two parton lines. We neglect all convolutions with number of the intermediate states greater than two. It has no impact on our further analysis because every convolution should be finite independently of the multiplicity of intermediate states. Exploiting the IR stability of the convolution we convert it into the basic QCD convolution and to the K_T - factorization. This transition is performed with purely mathematical means. In contrast, the transition from the K_T - to the collinear factorization is based on the physical assumption: we assume that the k_\perp - dependence of the parton distribution has one or several sharp maximums which become the intrinsic factorization scales. The sharper the maximums are, the more accurate this reduction is. In order to keep the lower blob unperturbative, the value of the intrinsic scale(s) should be close to Λ_{QCD} . Our assumption of the peaked k_\perp - distributions can be checked by analysis of experimental data in the framework of the k_T -factorization. Transition to the conventional parton distributions ϕ defined at other factorization scales μ located in the domain of the perturbative QCD (conventionally $\mu \sim$ several GeV), can be done with the use of the evolution equations. On the other hand, the perturbative scale can be regarded as the one achieved with the perturbative evolution starting from a lower scale which can be associated with our intrinsic scale μ_0 . Therefore, the conventional approach involves the intrinsic scale, though implicitly, while our approach sets this scale explicitly.

VII. ACKNOWLEDGEMENT

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